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The Little–Parks effect and multiquanta vortices in a hybrid superconductor–ferromagnet system

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Abstract
Within the phenomenological Ginzburg–Landau theory we investigate the phase diagram of a thin superconducting film with ferromagnetic nanoparticles. We study the oscillatory dependence of the critical temperature on an external magnetic field, similar to the Little–Parks effect, and formation of multiquantum vortex structures. The structure of a superconducting state is studied both analytically and numerically.

The Little–Parks effect [1], i.e. oscillations of the critical temperature T_c of multiply connected superconducting samples in an applied magnetic field H, is one of the striking phenomena demonstrating the coherent nature of the superconducting state. Such oscillatory behaviour of $T_c(H)$ is not specific to superconducting thin wall cylinders and can be observed also in superconductors with columnar defects and holes (see [2–4]) and mesoscopic simply connected samples of the size of several coherence lengths [5, 6]. Generally, the oscillations of T_c with change in the external magnetic flux are caused by the transitions between the states with different vorticities (winding numbers) characterizing the circulation of the phase of the order parameter. For a system with cylindrical symmetry the vorticity parameter just coincides with the angular momentum of the Cooper pair wavefunction. The states with a certain angular momentum m can be considered as m-quanta vortices. Experimental and theoretical investigations of these exotic vortex structures (multiquanta vortices and vortex molecules) in mesoscopic superconductors have attracted a great deal of attention. As we change an external homogeneous magnetic field, multiquanta vortices and vortex molecules can transform into one another via first or second order phase transitions.

In this paper we focus on another possibility for creating multiquantum vortex states: nucleation of the superconducting order parameter in a hybrid system consisting of a thin superconducting film and an array of magnetic nanoparticles. The interest in such structures is stimulated by their large potential for applications (e.g., as switches or systems with a controlled artificial pinning). Enhancement of the depinning critical current density j_c has been observed experimentally for superconducting films with arrays of submicron magnetic dots [7–9] and antidots [10], and for superconductor–ferromagnet (S/F) bilayers with domain structure in ferromagnetic films [11]. The matching effects observed for magnetic and transport characteristics were explained in terms of commensurability between the flux lattice and the lattice of magnetic particles. Vortex structures and pinning in the S/F systems at rather low magnetic fields (in the London approximation) have been analysed in the papers [12–21].

Provided that the thickness of a superconducting film is rather small as compared with the coherence length, the critical temperature of the superconducting transition as well as the structure of the superconducting nuclei should be determined by a two dimensional distribution of a magnetic field component $B_z(x, y)$ (perpendicular to the superconducting film plane) induced by the ferromagnetic particles. Obviously, the highest critical temperature corresponds to the nuclei which appear near the lines of zeros of B_z due to a mechanism analogous to the one responsible for the surface superconductivity (see, e.g., [22]) and domain wall superconductivity [23–25]. Provided that these lines of zeros have the shape of closed loops, the winding number of a superconducting nucleus will be determined by the magnetic flux through the loop. Thus, changing this flux (e.g., increasing an external H field applied along the z axis) we can control the winding number. The resulting phase transitions between the multiquantum states with different m can cause the oscillations of T_c. Such oscillatory behaviour has been, in fact, observed in [26] for a Nb film with an array of GdCo particles. Note that a change in the slope of the phase transition curve $T_c(H)$ (which is probably a signature of the transition discussed above) has also been found in [27] for a Pb film with CoPd particles. Provided that the dimensions of the sample in the (xy) plane are compared with the coherence length, we can expect a rather complicated picture which is influenced both by the sample edges and by the distribution of an inhomogeneous magnetic field. Recently oscillatory behaviour of $T_c(H)$ has been observed experimentally in a mesoscopic Al disc with a single magnetic dot [28]. For several model profiles of the magnetic field the resulting phase transitions between different types of exotic vortex state in a mesoscopic disc have been studied numerically in [29]. The interplay between the boundary effects and magnetic field inhomogeneity also influences the formation of multiquantum vortex states around a finite size magnetic dot embedded in a large area superconducting film [30, 31]. The transitions between different multiquantum vortex states with change in magnetic field and magnetic dot parameters were studied in [29–31] for certain temperature values. These effects are closely related to the ones observed in mesoscopic and multiply connected samples, and, consequently, we can expect the oscillations of $T_c(H)$ (analysed below) also to be a common feature to multiply connected superconductors and thin film systems with magnetic dots.

In this work we do not consider the magnetic phase transitions in the mixed state for $T < T_c$ and focus on the oscillatory behaviour of $T_c(H)$ in a large area superconducting film caused only by the quantization associated with the characteristics of the inhomogeneous magnetic field produced by ferromagnetic particles. We neglect the influence of the edge and proximity effects in the S/F system and consider a nanoparticle only as a source of a small scale magnetic field. Our further consideration is based on the linearized Ginzburg–Landau model:

$$-\left(\nabla + \frac{2\pi \mathrm{i}}{\Phi_0}\mathbf{A}\right)^2 \Psi = \frac{1}{\xi^2(T)}\Psi. \tag{1}$$

Here $\Psi(\mathbf{r})$ is the order parameter, $\mathbf{A}(\mathbf{r})$ is the vector potential, $\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$, Φ_0 is the magnetic flux quantum, $\xi(T) = \xi_0/\sqrt{1 - T/T_{c0}}$ is the coherence length, and T_{c0} is the critical temperature of the bulk superconductor at $B = 0$. For the sake of simplicity we neglect the effects of interference between the superconducting nuclei appearing near different nanoparticles (i.e. we assume the interparticle distance to be rather large as compared with the superconducting nucleus size) and consider a single magnetic particle with a fixed magnetic moment chosen perpendicular to the film plane xy. For a rather thin film (of thickness less than the coherence length) we can neglect the influence of the field components B_x, B_y in the film plane and consider an axially symmetrical two dimensional problem (1) in the field $B_z(r) = H + b(r)$, where $b(r)$ is the z component of the field induced by the ferromagnetic particle and (r, θ, z) is a cylindrical coordinate system. Choosing the gauge $A_\theta(r) = Hr/2 + a(r)$ one can find the solution of the equation (1) in the form $\Psi(\mathbf{r}) = g_m(r)\exp(\mathrm{i}m\theta)/\sqrt{r}$, where m is the vorticity, and $g_m(r) \propto r^{|m|+1/2}$ for $r \to 0$. The function $g_m(r)$ should be determined from the equation

$$-\frac{\mathrm{d}^2 g_m}{\mathrm{d}r^2} + \left[\frac{(\Phi(r)/\Phi_0 + m)^2}{r^2} - \frac{1}{4r^2}\right] g_m = \frac{1}{\xi^2(T)} g_m. \qquad (2)$$

Here $\Phi(r) = 2\pi r A_\theta(r)$ is the total flux through the circle of radius r. The lowest eigenvalue $1/\xi^2(T)$ of the Schrödinger-like equation (2) defines the critical temperature T_c of the phase transition into a superconducting state. Note that the similar problem of the energy spectrum of two dimensional electronic gas for a specific inhomogeneous magnetic field profile (i.e., the field of a magnetic antidot) has been studied in [32].

Obviously, for rather small fields H the superconducting order parameter can nucleate either far from the magnetic particle ($r \to \infty$) where the critical temperature T_c^H is defined by the homogeneous field $B_z = H$ or in the region close to the circle of radius r_0 where $B_z(r_0) = 0$ and T_c is controlled by the slope of $B_z(r)$ at $r = r_0$ and by the flux through the area of the radius r_0. In the first case we obtain $1 - T_c^H/T_{c0} = 2\pi|H|\xi_0^2/\Phi_0$. For the second case we can analyse the behaviour of $T_c(H)$ assuming that the characteristic length scale ℓ of the order parameter nucleus is much less than the characteristic scale of the magnetic field distribution. Within such local approximation (similar to the one used in [25] for the description of domain wall superconductivity) we can expand the flux in powers of the distance from r_0:

$$\frac{\Phi(r)}{\Phi_0} + m \simeq \left(\frac{\Phi(r_0)}{\Phi_0} + m\right) + \frac{\pi r_0 B_z'(r_0)}{\Phi_0}(r - r_0)^2.$$

This local approximation is valid under the following conditions:

$$\left|\frac{B_z''(r_0)}{B_z'(r_0)}\ell\right| \ll 1 \qquad \text{and} \qquad \frac{\ell}{r_0} \ll 1.$$

Introducing a new coordinate $t = (r - r_0)/\ell$ we obtain the dimensionless equation

$$-\frac{\mathrm{d}^2 g}{\mathrm{d}t^2} + (t^2 - Q)^2 g = Eg, \qquad (3)$$

where the parameters E and Q are given by the expressions

$$E = \frac{\ell^2}{\xi_0^2}\left(1 - \frac{T}{T_{c0}}\right), \qquad \ell = \sqrt[3]{\frac{\Phi_0}{\pi|B_z'(r_0)|}}, \qquad Q = -\left(\frac{\Phi(r_0)}{\Phi_0} + m\right)\sqrt[3]{\frac{\Phi_0}{\pi r_0^3 B_z'(r_0)}}. \qquad (4)$$

We obtain $E(Q) \simeq Q^2 + \sqrt{-2Q}$ when $Q \ll -1$, and $E(Q) \simeq 2\sqrt{Q}$ when $Q \gg 1$. The minimal value of $E(Q)$ is $E = E_{\min} \simeq 0.904$ at $Q \simeq 0.437$. The final expression for the critical temperature reads

$$1 - \frac{T_c}{T_{c0}} = \frac{\xi_0^2}{\ell^2}\left[\min_m E\left(-\left(\frac{\Phi(r_0)}{\Phi_0} + m\right)\sqrt[3]{\frac{\Phi_0}{\pi r_0^3 B_z'(r_0)}}\right) + \mathrm{O}\left(\frac{\ell^2}{r_0^2}\right)\right]. \qquad (5)$$

The superconducting nuclei are localized near the ferromagnetic particle at a distance r_0. The states with different energetically favourable winding numbers m correspond to multiquantum vortex structures very similar to the ones observed in a mesoscopic disc. As we change an external field H, we change the flux $\Phi(r_0)$ and, thus, change the energetically favourable vorticity number and position of the nucleus.

To investigate the details of the oscillatory behaviour discussed above we consider a particular case of a small ferromagnetic particle which can be described as a point magnetic dipole with a magnetic moment $\mathbf{M} = M\mathbf{z}_0$ placed at a height h over the superconducting film. The corresponding expressions for the field and the vector potential are

$$b(r) = \frac{M(2h^2 - r^2)}{(r^2 + h^2)^{5/2}}, \qquad a(r) = \frac{Mr}{(r^2 + h^2)^{3/2}}. \tag{6}$$

Introducing $f_m(r) = g_m(r)/\sqrt{r}$ and a dimensionless coordinate $\rho = r/h$ we obtain the equation (2) in the form

$$-\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho\frac{\mathrm{d}f_m}{\mathrm{d}\rho}\right) + \left(\frac{3\sqrt{3}}{2}N_\mathrm{f}\left[\frac{H}{b_0}\rho + \frac{\rho}{(1+\rho^2)^{3/2}}\right] + \frac{m}{\rho}\right)^2 f_m = \frac{h^2}{\xi_0^2}\left(1 - \frac{T_\mathrm{c}}{T_{\mathrm{c}0}}\right) f_m, \tag{7}$$

where $N_\mathrm{f} = 4\pi M/(3\sqrt{3}h\Phi_0)$ is the dimensionless flux through the area with the positive field $b(r)$ and $b_0 = b(0) = 2M/h^3$. In the limit of small fields $H \to 0$ the nucleation of superconductivity occurs at large distances ρ, and critical temperatures for different winding numbers m are very close. Thus, in this limit the critical temperature is equal to T_c^H and is not sensitive to the presence of the dipole. Below this temperature we obtain a lattice of singly quantized vortices (with the concentration determined by H) which is surely disturbed under the dipole. Note, that the behaviour of T_c in this low field regime should modify provided that we take account of the finite distance between the magnetic particles. For large absolute values $|H|$ (much larger than the maximum field induced by the dipole) we obtain the following asymptotical behaviour of T_c: $1 - T_\mathrm{c}/T_{\mathrm{c}0} = 2\pi\xi_0^2(-H - b_0)/\Phi_0$ for negative H and $1 - T_\mathrm{c}/T_{\mathrm{c}0} = 2\pi\xi_0^2(H - b_0/(25\sqrt{5}))/\Phi_0$ for positive H values (here $-b_0/(25\sqrt{5})$ is the minimum of the dipole field). The superconductivity nucleates near the minima of the total field $|B_z|$ and, thus, is localized near the dipole. In the intermediate field region $(-1 < H/b_0 < 1/(25\sqrt{5}))$ we should expect the oscillatory behaviour of T_c discussed above. The number of oscillations is controlled by the parameter N_f. We have carried out numerical calculations of equation (7) for various N_f values. For the numerical analysis of the localized states of equation (7) we approximated it on a equidistant grid and obtained the eigenfunctions $f_m(\rho)$ and eigenvalues by the diagonalization method of the tridiagonal difference scheme. The results of these calculations as well as the analytical dependence of T_c given by expression (5) are shown in figure 1. Typical profiles of the superconducting order parameter for different values of external magnetic field are given in figure 2.

We observed a remarkable asymmetry of the phase transition curve ($T_\mathrm{c}(H) \neq T_\mathrm{c}(-H)$) which is caused by the difference in distribution of positive and negative parts of the dipole field $b(r)$: the maximum positive field (b_0) is much larger than the absolute value of the minimum negative field ($b_0/(25\sqrt{5})$). As a result, the T_c oscillations appear to be most pronounced for negative H which compensates the positive part of the dipole field. Taking $M \sim 3\times10^{-11}$ G cm^3 (for a ferromagnetic particle with dimensions 300 nm$\times$300 nm$\times$300 nm and magnetization $\sim10^3$ G), $h \sim 300$ nm, we obtain $N_\mathrm{f} \simeq 10$, $b_0 \sim 10^3$ G, and the characteristic scales of T_c oscillations $\Delta H \sim 100$ Oe, $\Delta T_\mathrm{c} \sim 10^{-2}T_{\mathrm{c}0} \sim 0.1$ K for a Nb film with $\xi_0 \sim 40$ nm and $T_{\mathrm{c}0} \sim 8$ K.

We expect the oscillatory behaviour of T_c to be observable, e.g., in magnetoresistance measurements on thin superconducting films with arrays of ferromagnetic particles. The superconducting nuclei localized near the particles should result in a partial decrease in the resistance below the oscillating $T_\mathrm{c}(H)$. As we decrease the temperature below $T_\mathrm{c}(H)$

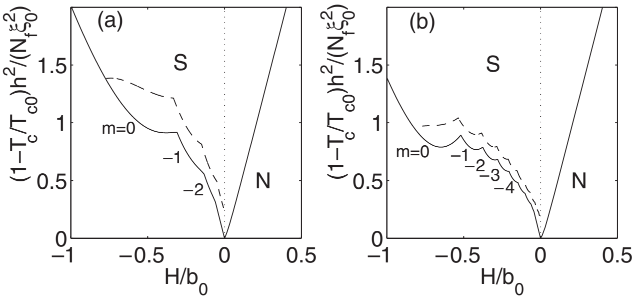

Figure 1. Critical temperature as a function of external magnetic field for $N_{\mathrm{f}} = 4$ (a) and $N_{\mathrm{f}} = 10$ (b). The solid curve is a result from direct numerical simulations of equation (7). The dashed curve is obtained from the analytical formula (5). Certain winding numbers m for different parts of the phase transition line are shown.

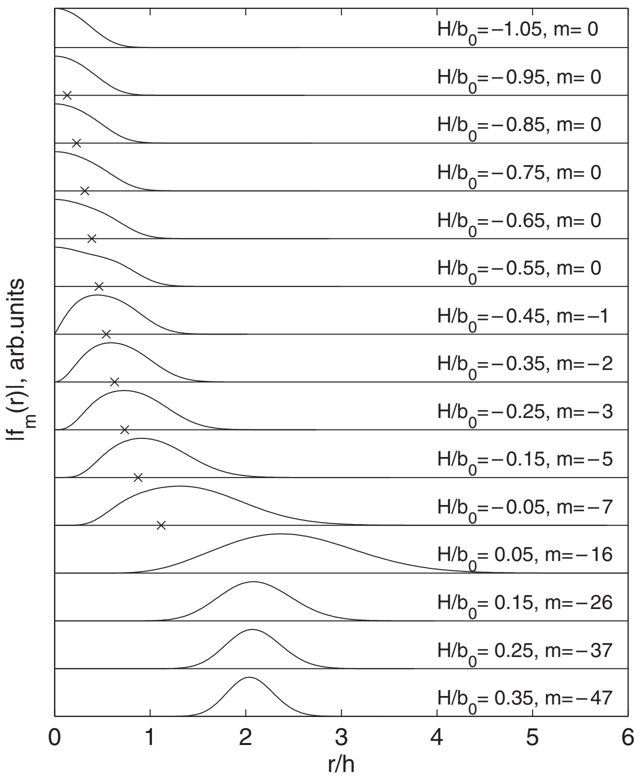

Figure 2. Typical spatial profiles of the order parameter absolute value for different external field values H for $N_{\mathrm{f}} = 10$. The cross ($\times$) marks the point where the total magnetic field B_z for a given plot is zero.

the superconducting order parameter around a single particle becomes a mixture of angular harmonics with different m values and we can expect the appearance of phase transitions similar to the ones discussed in papers [29, 31]. With further decrease in temperature the whole film becomes superconducting and the resistivity becomes zero.

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